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NONSTANDARD LORENTZ SPACE FORMS

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In their recent paper [8], Kulharni and Raymond show that a closed 3-manifold which admits a complete Lorentz metric of constant curvature 1 (henceforth called a *complete Lorentz structure*) must be Seifert fibered over a hyperbolic base. Furthermore on every such Seifert fibered 3-manifold with nonzero Euler class they construct such a Lorentz metric. Moreover the Lorentz structure they construct has a rather strong additional property, which they call "standard": A Lorentz structure is *standard* if its causal double cover possesses a timelike Killing vector field. Equivalently, it possesses a Riemannian metric locally isometric to a left-invariant metric on $\mathrm{SL}(2,\mathbf{R})$. Kulkarni and Raymond asked if every closed 3-dimensional Lorentz structure is standard. This paper provides a negative answer to this question (Theorem 1) and a positive answer to the implicit question raised in [8, 7.1.1] (Theorem 3).

Theorem 1. *Let M^3 be a closed 3-manifold which admits a homogeneous Lorentz structure and satisfies $H^1(M; \mathbf{R}) \neq 0$. Then there exists a nonstandard complete Lorentz structure on M.*

In [8] it is shown that the unit tangent bundle of a closed surface admits a homogeneous Lorentz structure. Therefore we obtain:

Corollary 2. *There exists a complete Lorentz structure on the unit tangent bundle of any closed surface F of genus greater than one which is not standard.*

The homogeneous Lorentz structures are all classified in [8]. A circle bundle of Euler number j over a closed surface F, $\chi(F) < 0$, has a homogeneous structure if and only if $j|\chi(F)$ (an analogous statement holds when M has singular fibers, i.e. when F is an orbifold).

We also show:

Theorem 3. *Let M^3 be a 3-manifold which admits a complete Lorentz structure. Then M^3 is not covered by a product $F \times S^1$, F a closed surface.*

Theorem 3 implies that the Euler class of the Seifert fiber structure of M^3 is *nonzero*.

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