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CONFORMAL DEFORMATIONS OF COMPLETE MANIFOLDS WITH NEGATIVE CURVATURE

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Introduction

A basic problem in Riemannian geometry is that of studying the set of curvature functions that a manifold possesses. In this generality there has been such a great deal of work that we cannot here record the different contributions. (For a fairly complete account, see [23].) However, in this paper we shall be concerned with the special case of ("pointwise") conformal deformations of metrics which we shall call problem (κ):

> Let M be an n-dimensional Riemannian manifold with metric g. If we are given a real-valued function on M, does there exist a metric $\tilde{g}$ on M, conformal to g, with the given function as its curvature (i.e., Gaussian curvature if $n = 2$, and scalar curvature if $n \geqslant 3$)?

This problem has been extensively studied for compact manifolds with or without boundary (see [6], [7], [9], [13], [14], [15], and [18]). However there are still some unsettled questions, even for $M = S^2$ with the standard metric (see [18]), or on more general manifolds. The special case of deforming to constant scalar curvature is known as the Yamabe Problem and has recently been completely resolved for compact manifolds by R. Schoen [21] (see also [6]).

On the other hand, if M is a complete but noncompact Riemannian manifold, very little is known. In the special case $M = \mathbf{R}^n$ with Euclidean metric g, problem (κ) has been studied in [4], [16], [17], [19], and [20]. The purpose of this paper is to study (κ) for simply-connected manifolds with negative curvature. The model case is $H^n(-1)$, the n-dimensional space form of curvature -1, and Kazdan has posed (κ) for $H^n(-1)$ and more general manifolds of negative curvature as an open problem in [12].

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