J. DIFFERENTIAL GEOMETRY
21 (1985) 231–268

3-DIMENSIONAL LORENTZ SPACE-FORMS AND SEIFERT FIBER SPACES

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1. Introduction

A *space-form* is a complete pseudo-Riemannian manifold of dimension $\geqslant 2$ with constant curvature. A *Lorentz space-form* is a space-form with a Lorentz metric of signature $+--\cdots$. In this paper we study 3-dimensional Lorentz space-forms of constant curvature 1, and unless there is a possibility of confusion, these will be often referred to simply as space-forms. The standard linear model for this geometry (the "3-dimensional anti-de Sitter space") is

$$S^{1,2} = \{(x, y) | x, y \in \mathbf{R}^2, |x|^2 - |y|^2 = 1\} \approx O(2,2)/O(1,2),$$

cf. [38, p. 334]. This set-up differs markedly from the usual Riemannian set-ups in two respects: (1) the isotropy subgroup $O(1,2)$ is noncompact, so $O(2,2)$ *does not act properly* on $S^{1,2}$. This feature substantially restricts the discrete subgroups of $O(2,2)$ which can act properly discontinuously on $S^{1,2}$. (2) On

Received November 28, 1983 and, in revised form, January 10, 1985. The first author was partially supported by an NSF grant and a Guggenheim fellowship, and the second author by an NSF grant.