J. DIFFERENTIAL GEOMETRY
21 (1985) 109–127

A COMPLETE EMBEDDED MINIMAL SURFACE IN $\mathbf{R}^3$ WITH GENUS ONE AND THREE ENDS

DAVID A. HOFFMAN[1] & WILLIAM MEEKS III

1. Introduction

It has been a longstanding conjecture that the only complete embedded minimal surfaces in $\mathbf{R}^3$ of finite topological type are the plane, the catenoid, and the helicoid. This conjecture is false. We will exhibit a complete minimal surface, conformally the square torus $\mathbf{C}/\mathbf{Z}^2$ with three points removed, which is embedded in $\mathbf{R}^3$. It has the following interesting geometric properties.

- Its Gauss map composed with stereographic projection is given by a constant divided by the derivative of the Weierstrass P-function.
- It contains two straight lines which meet at right angles.
- It can be decomposed into eight congruent pieces, each of which lies in a different octant and each of which is a graph.
- It is invariant under the group of motions of $\mathbf{R}^3$ generated by

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(which is the dihedral group with eight elements).

This surface was first written down by Costa in his thesis [3]. He established that it was complete, and of genus one with three ends (see Theorem 1). We computed the coordinates of the surface and drew computer pictures of it. Observing that it looked embedded and it had dihedral symmetry, we were

---

Received April 3, 1985.
[1]The first author was partially supported by NSF Grant MCS-83-01936, and the second author partially supported by NSF Grant DMS-84-14330.