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RICCI DEFORMATION OF THE METRIC ON A RIEMANNIAN MANIFOLD

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The interaction between algebraic properties of the curvature tensor and the global topology and geometry of a Riemannian manifold has been studied extensively. Of particular interest is the question under which conditions on its curvature tensor a Riemannian manifold is homeomorphic or diffeomorphic to a space of constant positive sectional curvature.

A Riemannian manifold M with sectional curvature κ is said to be δ-pinched if $\delta < \kappa \leqslant 1$ holds globally on M. The famous sphere theorem then states that a complete, simply connected $\frac{1}{4}$-pinched manifold is homeomorphic to the standard sphere [1], [6], [7]. It is also known that the homeomorphism theorem can be sharpened to a diffeomorphism theorem, if a more restrictive pinching condition is imposed, [3], [8]. Recently, Ruh [9] was able to show with the help of the Calderon-Zygmund inequalities, that the global pinching condition can be weakened to a local one: If the curvature ratios of a compact Riemannian manifold of positive sectional curvature are close to one, then the manifold is diffeomorphic to a spherical space form.

In this paper we use the heat flow method developed by Hamilton in [4] to give a new proof of Ruh's result and to obtain a more precise pointwise condition for the curvature tensor which ensures the existence of a diffeomorphism to a spherical space form. In [4] Hamilton showed that on a three-dimensional manifold of strictly positive Ricci curvature the metric can be deformed into a metric of constant positive curvature. We show that this heat flow method works for any dimension $n \geqslant 4$, provided the norm of the Weyl conformal curvature tensor and the norm of the traceless Ricci tensor are not too large compared to the scalar curvature at each point.

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