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BOUNDS ON THE VON NEUMANN DIMENSION OF L^2-COHOMOLOGY AND THE GAUSS-BONNET THEOREM FOR OPEN MANIFOLDS

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0. Introduction

In this paper we continue the discussion of [6]. In §§1–3, we prove those results concerning the Von Neumann dimension of L^2-cohomology spaces (L^2 Betti numbers) whose proofs were given only for normal coverings which are profinite.[1]

The L^2-cohomology techniques turn out to be useful in other contexts as well. For example, in [7] we simplify the proof of the theorem of Gottlieb and Stallings, which states that if a $K(\pi, 1)$-space is homotopy equivalent to a finite complex and π has nontrivial center, then the Euler characteristic $\chi(K(\pi, 1))$ vanishes. In fact, we show that it suffices to assume that π has a nontrivial normal amenable subgroup.

In §4 we extend the result of [6] concerning the η-invariant to the not necessarily profinite case. For this we define a corresponding invariant $\tilde{\eta}_{(2)}$ by means of the Γ-trace.

In §5 we extend our results to certain metrics which are conformally related to those considered in §1. We also give an intrinsic criterion (Theorem 5.5) for a metric to be of this type.

As background for §§1–3 of the present paper, we now recall some material from [6]. There we considered a complete riemannian manifold M^n, whose sectional curvature, K, and volume, $\mathrm{Vol}(M)$, satisfy $|K| \leqslant 1$, $\mathrm{Vol}(M) < \infty$. Here we will be concerned exclusively with the particular case in which

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[1] A covering $\tilde{M}$ is *profinite* if there exist subgroups $\Gamma_j \subset \pi_1(M)$, of finite index, such that $\bigcap \Gamma_j = \Gamma = \pi_1(\tilde{M})$.