1. Introduction

The early part of the 1980s has experienced a vast increase in our understanding of smooth 4-manifolds. This has been accomplished principally through the work of S. Donaldson, namely

**Theorem 1.1 (Donaldson [3]).** Let $M$ be a smooth closed oriented simply-connected 4-manifold with positive definite intersection form $\theta$. Then $\theta$ is “standard”, i.e. over the integers $\theta \equiv (1) \oplus \cdots \oplus (1)$.

Although this is a theorem about 4-dimensional topology, its proof is differential geometric and analytic. The main theme of Donaldson’s work is to study the topology of the space of solutions of the self-dual Yang-Mills equations on an $SU(2)$-bundle over the Riemannian manifold $M$ and relate it to the topology of $M$. Recently we attempted to apply these Yang-Mills techniques to problems we have been working on for several years; namely finding numerical invariants for homology 3-spheres bounding acyclic 4-manifolds and studying smooth pseudo-free circle actions on 5-manifolds. We were moderately successful in [4]. That work studied Yang-Mills equations invariant under a cyclic group action. However, we were unsuccessful in following the complete “Donaldson program” in this equivariant setting. In particular, we were unable to mimic the work of C. Taubes [10] in finding nicely parameterized solutions to the Yang-Mills equations and were forced to use ad hoc, less analytical techniques. We then “unequivariantized” our proof only to realize that we had a “simpler” proof of a version of Donaldson’s theorem under the weaker (more topologically useful) assumption that $H_1(M; \mathbb{Z})$ has no 2-torsion.

The goals of this paper are, then, to give a proof of a version of Donaldson’s theorem which on the one hand is more accessible to topologists and on the