J. DIFFERENTIAL GEOMETRY
20 (1984) 479–495

CONFORMAL DEFORMATION OF A RIEMANNIAN METRIC TO CONSTANT SCALAR CURVATURE

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A well-known open question in differential geometry is the question of whether a given compact Riemannian manifold is necessarily conformally equivalent to one of constant scalar curvature. This problem is known as the Yamabe problem because it was formulated by Yamabe [8] in 1960. While Yamabe's paper claimed to solve the problem in the affirmative, it was found by N. Trudinger [6] in 1968 that Yamabe's paper was seriously incorrect. Trudinger was able to correct Yamabe's proof in case the scalar curvature is nonpositive. Progress was made on the case of positive scalar curvature by T. Aubin [1] in 1976. Aubin showed that if dim $M \geqslant 6$ and M is not conformally flat, then M can be conformally changed to constant scalar curvature. Up until this time, Aubin's method has given no information on the Yamabe problem in dimensions 3, 4, and 5. Moreover, his method exploits only the local geometry of M in a small neighborhood of a point, and hence could not be used on a conformally flat manifold where the Yamabe problem is clearly a global problem. Recently, a number of geometers have been interested in the conformally flat manifolds of positive scalar curvature where a solution of Yamabe's problem gives a conformally flat metric of constant scalar curvature, a metric of some geometric interest. Note that the class of conformally flat manifolds of positive scalar curvature is closed under the operation of connected sum, and hence contains connected sums of spherical space forms with copies of $S^1 \times S^{n-1}$.

In this paper we introduce a new global idea into the problem and we solve it in the affirmative in all remaining cases; that is, we assert the existence of a positive solution u on M of the equation

$$(0.1) \qquad \Delta u - \frac{n-2}{4(n-1)} R u + u^{(n+2)/(n-2)} = 0,$$

Received September 13, 1984.