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KOSZUL COHOMOLOGY AND THE GEOMETRY OF PROJECTIVE VARIETIES. II

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0. Introduction

This paper continues the search begun in [2] for some new techniques to use in computing Koszul cohomology. The same notation will be used as in that paper.

One extremely natural question is to determine the Koszul cohomology groups $\mathscr{K}_{p,q}(\mathbf{P}^r, H^k, H^d, W)$, where $H \to \mathbf{P}^r$ is the hyperplane bundle, $d \geqslant 1$ and $W \subseteq H^0(\mathbf{P}^r, \mathcal{O}_{\mathbf{P}^n}(d))$ is a base-point free linear system. The simplest case of this is to ask when the multiplication map

$$(0.1) \qquad W \otimes H^0(\mathbf{P}^r, \mathcal{O}(k)) \to H^0(\mathbf{P}^r, \mathcal{O}(k+d))$$

must be surjective. Indeed, the surjectivity of (0.1) comes up in a conjecture of Carlson, Green, Griffiths & Harris [1]. Let

$$S_k \subset \mathbf{P}_{\binom{d+3}{3}-1}$$

be the variety of smooth surfaces in $\mathbf{P}^3$ of degree d which contain a curve C of degree k which is not a complete intersection. Is

$$(0.2) \qquad \operatorname{codim} S_k \geqslant d-3?$$

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