J. DIFFERENTIAL GEOMETRY
20 (1984) 237–266

FLOW BY MEAN CURVATURE OF CONVEX SURFACES INTO SPHERES

GERHARD HUISKEN

1. Introduction

The motion of surfaces by their mean curvature has been studied by Brakke [1] from the viewpoint of geometric measure theory. Other authors investigated the corresponding nonparametric problem [2], [5], [9]. A reason for this interest is that evolutionary surfaces of prescribed mean curvature model the behavior of grain boundaries in annealing pure metal.

In this paper we take a more classical point of view: Consider a compact, uniformly convex n-dimensional surface $M = M_0$ without boundary, which is smoothly imbedded in $\mathbf{R}^{n+1}$. Let M_0 be represented locally by a diffeomorphism

$$F_0: \mathbf{R}^n \supset U \to F_0(U) \subset M_0 \subset \mathbf{R}^{n+1}.$$

Then we want to find a family of maps $F(\cdot, t)$ satisfying the evolution equation

$$(1) \qquad \begin{aligned} &\frac{\partial}{\partial t} F(\vec{x}, t) = \Delta_t F(\vec{x}, t), \qquad \vec{x} \in U, \\ &F(\cdot, 0) = F_0, \end{aligned}$$

where Δ_t is the Laplace-Beltrami operator on the manifold M_t, given by $F(\cdot, t)$. We have

$$\Delta_t F(\vec{x}, t) = -H(\vec{x}, t) \cdot \nu(\vec{x}, t),$$

where $H(\cdot, t)$ is the mean curvature and $\nu(\cdot, t)$ is the outer unit normal on M_t. With this choice of sign the mean curvature of our convex surfaces is always positive and the surfaces are moving in the direction of their inner unit normal. Equation (1) is parabolic and the theory of quasilinear parabolic differential equations guarantees the existence of $F(\cdot, t)$ for some short time interval.

Received April 28, 1984.