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MODULI SPACE OF STABLE CURVES FROM A HOMOTOPY VIEWPOINT

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Dedicated to Daniel Mostow on his 60th birthday

1. Introduction

(1.1) Let J: $R(S) \to \mathfrak{S}_g/\mathrm{SP}_g(\mathbb{Z})$ be the period mapping of the Riemann space $R(S)$ of a nonsingular curve S of genus g to the Siegel modular space $\mathfrak{S}_g/\mathrm{SP}_g(\mathbb{Z})$ of degree g. Both of these spaces $R(S)$ and $\mathfrak{S}_g/\mathrm{SP}_g(\mathbb{Z})$ can be compactified to projective varieties in a natural manner. The compactification $\hat{R}(S)$ of $R(S)$ is known as the *moduli space of stable curves* or the *augmented Riemann space*. As for the Siegel space, its compactification $\mathfrak{S}_g^*/\mathrm{SP}_g(\mathbb{Z})$ is called the *Satake space*, and in our previous paper [7] we studied the stable cohomology $H^*(\mathfrak{S}_g^*/\mathrm{SP}_g(\mathbb{Z}))$ of this space. From the work of Namikawa (see [17]) it is known that the classical period mapping can be extended to a map J: $\hat{R}(S) \to \mathfrak{S}_g^*/\mathrm{SP}_g(\mathbb{Z})$ of the compactifications. One of our original goals in writing this paper was to study the cohomological nature of this map. Our result follows (see also (7.1.4)).

Theorem. *The stable cohomology $H^*(\mathfrak{S}_g^*/\mathrm{SP}_g(\mathbb{Z}); \mathbb{Q})$ of the Satake space is a tensor product of two polynomial rings $\mathbb{Q}[x_i] \otimes \mathbb{Q}[y_j]$, degree $x_i = 4i + 2$, $0 \leqslant i < \infty$, degree $y_j = 4j + 2$, $0 < j < \infty$. The induced map on cohomology J^*: $H^*(\mathfrak{S}_g^*/\mathrm{SP}_g(\mathbb{Z}); \mathbb{Q}) \to H^*(\hat{R}(S); \mathbb{Q})$ kills the second polynomial ring $J^*(y_{4j+2}) = 0$.*

Recently E. Miller proved independently that J^* maps the first polynomial ring $\mathbb{Q}[x_i]$ injectively into $H^*(R(S); \mathbb{Q})$ in a stable range (see [16]). Thus his results, combined with the above theorem provide a complete answer for the induced mapping of J on stable cohomology.

(1.2) To establish our theorem, we have to overcome some of the technical difficulties which are typical in studying the homotopy nature of moduli

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