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CONTRACTIONS OF INVARIANT FINSLER FORMS ON THE CLASSICAL DOMAINS

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Abstract

The Schwarz-Pick inequality shows that every holomorphic map of the complex unit disk into itself contracts the Poincaré metric. We consider the analogous question for holomorphic maps and biholomorphically invariant Finsler forms, on any of the classical Cartan domains of rank > 1.

Theorem. *Suppose D is a classical Cartan symmetric domain of rank > 1. If f: $D \to D$ is a nonconstant holomorphic map that contracts all invariant Finsler forms, then f is a biholomorphism.*

The question of which maps contract all invariant Finsler forms contrasts with previous work giving a Schwarz-Pick inequality for the Bergman metric on bounded symmetric domains (see e.g., Kobayashi: *Hyperbolic manifolds and holomorphic mappings*) and also with work defining systems of metrics contracted by all holomorphic mappings (the "Schwarz-Pick" systems of pseudometrics of Harris).

1. Introduction

Let D be any one of the four classical Cartan domains in C^n. This paper answers the following question: which holomorphic maps f: $D \to D$ contract all biholomorphically invariant infinitesimal Finsler forms on D?

If D is the Poincare disk then of course the answer is familiar. The infinitesimal Poincare metric is (up to scalar multiples) the only invariant infinitesimal Finsler form on D and by the Schwarz-Pick inequality every holomorphic f: $D \to D$ is a contraction.

When the rank of D is greater than 1 the situation is radically different. We shall see that except for the biholomorphisms themselves, there are *no* nonconstant holomorphic contractions of every invariant Finsler form.

The question of which maps contract every invariant form seems to contrast with previous work on the Schwarz-Pick inequality. Koranyi [6] showed that if G is a bounded symmetric domain of rank k then every holomorphic f: $G \to G$

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