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FUNCTION THEORY, RANDOM PATHS AND COVERING SPACES

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0. Introduction

For connected Riemannian manifolds M we discuss the interplay between the harmonic function theory on M, the statistical properties of random paths on M and the global geometrical structure of M.

In particular, we study the case when M is a regular or Galois cover of a smaller Riemannian manifold N. That is, there is a discrete group Γ of isometries acting on M so that $N = M/\Gamma$. M will be called an Abelian (resp. nilpotent, solvable, etc.) cover of N when Γ is an Abelian (nilpotent, solvable, etc.) discrete group of isometries.

We first illustrate the general results by an example. Let M be any Abelian cover of any compact Riemann surface N (the metric chosen for N is of no significance). Let the genus of N exceed 1 and the rank of the Abelian group exceed 2. Then by Theorems 1 and 4 below one sees that:

(i) M does not possess any nonconstant positive harmonic functions, *but*

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