

## FUNCTION THEORY, RANDOM PATHS AND COVERING SPACES

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### 0. Introduction

For connected Riemannian manifolds  $M$  we discuss the interplay between the harmonic function theory on  $M$ , the statistical properties of random paths on  $M$  and the global geometrical structure of  $M$ .

In particular, we study the case when  $M$  is a regular or Galois cover of a smaller Riemannian manifold  $N$ . That is, there is a discrete group  $\Gamma$  of isometries acting on  $M$  so that  $N = M/\Gamma$ .  $M$  will be called an Abelian (resp. nilpotent, solvable, etc.) cover of  $N$  when  $\Gamma$  is an Abelian (nilpotent, solvable, etc.) discrete group of isometries.

We first illustrate the general results by an example. Let  $M$  be any Abelian cover of any compact Riemann surface  $N$  (the metric chosen for  $N$  is of no significance). Let the genus of  $N$  exceed 1 and the rank of the Abelian group exceed 2. Then by Theorems 1 and 4 below one sees that:

- (i)  $M$  does not possess any nonconstant positive harmonic functions, *but*