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ON THE GAUSS MAP OF AN AREA-MINIMIZING HYPERSURFACE

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0. Introduction

Let S be an area-minimizing hypersurface of $\mathbf{R}^N$. By *hypersurface*, we mean a codimension one, locally integral current. By *area-minimizing*, we mean that, without introducing boundary, no compact piece of S can be replaced by a piece having less area. The main concern of this paper is the relationship between S and its spherical image under the Gauss map. For a more precise treatment of our terminology, here and below, the reader should refer to §1.

In the 1960's, area-minimizing hypersurfaces provided the focus for a great deal of research. Indeed, a major accomplishment of that period was the discovery that, upon imposing the dimensional restriction $N < 8$, one guarantees to S several very strong properties which, generally, fail to hold as soon as $N > 7$. Paramount among these properties are two: *interior regularity* and, its alter ego, the *parametric Bernstein property*. The former states that $\operatorname{spt}(S) \sim \operatorname{spt}(\partial S)$ is a codimension one, real analytic submanifold of $\mathbf{R}^N$; the latter, that if $\partial S = 0$, then each connected component of $\operatorname{spt}(S)$ is a hyperplane.

Our paper here introduces a new condition which guarantees both of these properties, independently of dimension. That is, we show that the dimensional hypothesis $N < 8$ can be *replaced* by a different condition, one which involves the topology and Gauss image of $\operatorname{spt}(S) \sim \operatorname{spt}(\partial S)$. We thereby obtain a regularity result (Theorem 3), and a parametric Bernstein result (Theorem 5), which for example, imply the following: *Suppose* $H^1(\operatorname{reg} S) = 0$ *and* $\operatorname{Gauss}(S) \subset S^{N-1}$ *omits a thickened great* S^{N-3}. *Then* $\operatorname{spt}(S) \sim \operatorname{spt}(\partial S)$ *is smooth, and if* $\partial S = 0$, *consists of affine hyperplanes.* A local version of this also holds, estimating curvature when $\partial S \neq 0$ (Theorem 4).

Like our results, our methods follow a pattern developed in the classical (dimensionally restricted) case. Recall that there, regularity is proved by induction on N. The inductive step requires that a smooth minimal hypersurface M in S^{N-1} be an equator if the cone $0 \times\!\!\!\!\times M$ is area-minimizing. That

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