

# KOSZUL COHOMOLOGY AND THE GEOMETRY OF PROJECTIVE VARIETIES

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## 0. Introduction

There are a number of interesting problems and results which involve being able to compute Koszul cohomology groups; for example, the local Torelli problem, understanding the canonical ring of a variety of general type, Petri’s work on the ideal of a special curve, Mumford’s projective normality theorem, and Donagi’s work on the global Torelli theorem for projective hypersurfaces.