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STABLE MINIMAL SURFACES IN EUCLIDEAN SPACE

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To my wife

1. Introduction

In this paper we study two-real dimensional minimal surfaces in Euclidean space which are stable, that is, which minimize area on each compact set up to second order. The earliest result for such surfaces is due to S. Bernstein [4], who proved that all complete minimal graphs in $\mathbf{R}^3$ (these are automatically stable) are planes. This Bernstein result was generalized by R. Osserman in [21] and [22] where he showed that any complete minimal surface in $\mathbf{R}^3$ whose Gauss map omits an open set (or even just a set of positive logarithmic capacity) from the sphere must be a plane. In these theorems Osserman makes no assumption about the stability of the minimal surface. The theorem of Osserman was later extended, partially by S. S. Chern [8] and to its full generality by Osserman [23], to minimal surfaces in $\mathbf{R}^n$, for any n, again without any assumption on stability. F. Xavier [30] has recently strengthened the theorem of Osserman for minimal surfaces in $\mathbf{R}^3$ in a remarkable way: he has proved that if the Gauss map of a complete minimal surface in $\mathbf{R}^3$ omits 7 or more points from the sphere, then it must be a plane. The relationship of the stable regions on a minimal surface M to the area of their Gaussian image has been studied by J. L. Barbosa and M. doCarmo in [3]. The methods of R. Schoen, L. Simon and S. T. Yau [25] yield a proof of the Bernstein result for stable minimal surfaces M in $\mathbf{R}^3$ provided the area growth of a ball of geodesic radius r in M is not larger than r^6. (This condition is automatically satisfied by all minimal graphs in $\mathbf{R}^3$.) A classification theorem for complete stable minimal surfaces in three dimensional manifolds of nonnegative scalar curvature has been obtained by D. Fischer-Colbrie and R. Schoen in [12]. A corollary of their theorem states that all complete oriented stable minimal surfaces in $\mathbf{R}^3$ are planes, thereby giving another direct generalization of the Bernstein theorem. This corollary was also proved by M. doCarmo and C. K. Peng [6].

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