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HILBERT STABILITY OF RANK-TWO BUNDLES ON CURVES

DAVID GIESEKER & IAN MORRISON

1. Let k be an algebraically closed field, and let d and g be two integers with $g \geqslant 2$ and $d \geqslant 1000g(g-1)$. Let $n = d + 2 - 2g$, and let W be a vector space of dimension n. G will denote the grassmannian of all codimension-two subspaces of W, and $\mathcal{E}$ will denote the universal rank-two bundle on G. In this paper, a curve will be a connected one-dimensional projective scheme. Let C be a curve on G, i.e., C is a subscheme of G which is a curve, and consider $E = \mathcal{E}_C = \mathcal{E}_{|C}$. Let $P_C(m) = \chi((\det E)^{\otimes m})$ be the Hilbert polynomial of C where $\det E = \wedge^2 E$. We let $S_{g,d}$ be the set of all curves C on G with $P_C(m) = dm + 2 - 2g$. Thus $S_{g,d}$ is the set of all curves of genus g and degree d on G.

Now W is identified with $H^0(G, \mathcal{E})$, so given $C \in S_{g,d}$, there is a natural map

$$\varphi_1 : W \to H^0(C, E).$$

We will identify W with $H^0(C, E)$ if φ_1 is an isomorphism. Thus we obtain a map

$$\varphi_2 : \wedge^2 W \to H^0(C, \wedge^2 E).$$

So for any positive integer m, we obtain a map

$$\varphi_3 : S^m(\wedge^2 W) \to H^0(C, (\det E)^{\otimes m}).$$

We may and do choose m so that φ_3 is onto, so that $h^0(C, (\det E)^{\otimes m}) = P_C(m)$ for any $C \in S_{g,d}$. Thus we finally obtain a map

$$\varphi_C^m : \bigwedge^{P_C(m)} S^m(\wedge^2 W) \to \bigwedge^{P_C(m)} H^0(C, (\det E)^{\otimes m}) \cong k.$$

We say $C \subseteq G$ is m-Hilbert stable (resp., m-Hilbert semistable) if φ_C^m is properly stable (resp., semistable) under the induced action of $SL(W)$ in the

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