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UNIQUENESS, SYMMETRY, AND EMBEDDEDNESS OF MINIMAL SURFACES

RICHARD M. SCHOEN

In 1956, A. D. Alexandrov [1] proved that a closed embedded hypersurface of constant mean curvature in Euclidean space is a standard sphere. Besides the importance of this result in differential geometry, the method employed in its proof has been used on a variety of problems in partial differential equations and differential geometry, first by J. Serrin [13] and more recently by B. Gidas, W. M. Ni and L. Nirenberg [2]. In a surprising recent development, W. Y. Hsiang, Zhen-Huan Teng and Wen-ci Yu [4] have constructed non-spherical closed immersed hypersurfaces of constant mean curvature in $\mathbf{R}^4$. These examples show that the embeddedness hypothesis is essential in Alexandrov's theorem. In this paper we apply Alexandrov's method to minimal hypersurfaces. The main difficulty, of course, is that minimal surfaces are never closed, but either have boundary or are complete and noncompact. An interesting feature of our results is that the embeddedness is not required; in fact, a consequence of the method is that in certain cases immersed surfaces can be shown to be embedded. This can be partially attributed to the fact that minimal hypersurfaces do not have a distinguished side locally whereas surfaces of nonzero constant mean curvature do.

Also in 1956, M. Shiffman [14] posed the problem of understanding minimal surfaces in $\mathbf{R}^3$ whose boundary consists of a union of two Jordan curves Γ_1, Γ_2 lying in parallel planes. Shiffman proved the striking result that if M is an immersed minimal surface of genus zero with $\partial M = \Gamma_1 \cup \Gamma_2$ and if Γ_1, Γ_2 are convex curves (resp. circles), then M meets each intermediate plane transversally in a convex curve (resp. circle). In particular this shows that if Γ_1 and Γ_2 are circles situated so that the line joining their centers is perpendicular to the planes in which they lie, then M is a surface of rotation, hence a catenoid. In §1 of this paper we extend this result in various directions; for example, we remove the topological assumption on M in the above characterization of the catenoid, and extend the results to higher dimensions. We also show that if Γ is

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