J. DIFFERENTIAL GEOMETRY
18 (1983) 669–699

VARIATIONAL PROBLEMS AND ELLIPTIC MONGE — AMPÈRE EQUATIONS

ILYA J. BAKELMAN

In the present paper we study the n-dimensional variational problems connected with the Dirichlet problem for n-dimensional Monge-Ampère equation

$$(*)\qquad \det\|u_{ij}\| = f(x_1, x_2, \cdots, x_n)$$

with zero boundary condition and prove the existence and uniqueness of an absolute minimum for this problem. This minimum is a generalized solution of the equation $(*)$ belonging to the class of all general convex functions.

The technique of convex hypersurfaces and bodies used in the geometric theory of elliptic Monge-Ampère equations turns out to be also essential for the investigations of the variational problems considered below. Therefore we also included the brief exposition of some necessary concepts and results of this theory.

CONTENTS

1. Introduction

1.1. Statement of problems. This paper is devoted to proving an existence and uniqueness of the absolute minimum for the functionals whose Euler equation is given by the Monge-Ampère equation

$$(1.1)\qquad \det\|u_{ij}\| = f(x_1, x_2, \cdots, x_n).$$

Received September 20, 1982, and, in revised form, February 24, 1983. This work was supported by NSF Grant MCS-8201106.