J. DIFFERENTIAL GEOMETRY
18 (1983) 659–668

# THE FUNDAMENTAL SOLUTION OF THE HEAT EQUATION ON A COMPACT LIE GROUP

H. D. FEGAN

## 1. Introduction

The purpose of this paper is to study the fundamental solution of the heat equation on a compact Lie group. Our main result is to express this function in terms of a product over the roots of the Lie group. The terms in this product are then identified as classical functions. The result is the following.

**Theorem 1.1.** *Let $G$ be a compact semisimple, simply connected Lie group. Then the fundamental solution of the heat equation is*

$$K(x,t)=\frac{\operatorname{vol} G e^{(2\mu+l)t/24}}{\pi^{2(2\mu+l)/3}2^{2(\mu+l)/3+\mu}}\left(-\theta'(t/8)\right)^{-(\mu-l)/3}$$
$$\times\prod_{\alpha>0}-\frac{\theta_3'(\pi\alpha(x)/2,it/8\pi)}{\sin\pi\alpha(x)}.$$

The notation in this theorem is the following. Firstly,

$$(1.1)\qquad \theta(t)=\sum e^{-n^2t}$$

with the sum over all integers and $\theta'(t)$ is the usual derivative of $\theta$. Then

$$(1.2)\qquad \theta_3'(z,t)=\frac{\partial\theta_3}{\partial z}(z,t)$$

where $\theta_3$ is the classical theta function of [5]. Notice that we are using $t$ for the second variable rather than $q=e^{i\pi t}$ which is used in [5]. The constant $\mu$ is the number of positive roots and $l$ is the rank of the Lie group.

The trace of the heat kernel is $K(1,t)$, where 1 is the identity element of the group. It follows immediately from Theorem 1.1 that we can express $K(1,t)$ in terms of classical functions.

**Corollary 1.2.** *The trace of the heat kernel is*

$$K(1,t)=\frac{\operatorname{vol} G e^{(2\mu+l)t/24}}{\pi^{2(2\mu+l)/3}2^{2(2\mu+l)/3}}\left(-\theta'(t/8)\right)^{(2\mu+l)/3}.$$

Received May 21, 1982.