

THE LOCAL STRUCTURE OF POISSON MANIFOLDS

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Historical Introduction

The classical Poisson bracket operation defined on functions on \mathbf{R}^{2n} is

$$(*) \quad \{f, g\} = \sum_{i,j=1}^n \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial g}{\partial q_i} \frac{\partial f}{\partial p_i} \right).$$

In the early nineteenth century, Poisson noticed that the vanishing of $\{f, g\}$ and $\{f, h\}$ imply that of $\{f, \{g, h\}\}$; almost thirty years later Jacobi discovered the identity $\{f, \{g, h\}\} = \{\{f, g\}, h\} + \{g, \{f, h\}\}$ which "explains" Poisson's theorem. In his study of general composition laws satisfying the Jacobi identity, Lie [29] defined in local coordinate form what is now known as a Poisson structure. On \mathbf{R}^r such a structure is given by functions $w_{ij}(x_1, \dots, x_r)$ satisfying the identities

$$w_{ij} + w_{ji} = 0,$$

$$\sum_{l=1}^r \left(w_{lj} \frac{\partial w_{ik}}{\partial x_l} + w_{li} \frac{\partial w_{kj}}{\partial x_l} + w_{lk} \frac{\partial w_{ji}}{\partial x_l} \right) = 0,$$

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