J. DIFFERENTIAL GEOMETRY
18 (1983) 523–557

THE LOCAL STRUCTURE OF POISSON MANIFOLDS

ALAN WEINSTEIN

Table of Contents

Historical Introduction

The classical Poisson bracket operation defined on functions on $\mathbf{R}^{2n}$ is

$$(*) \qquad \{f, g\} = \sum_{i,j=1}^{n} \left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial g}{\partial q_i}\frac{\partial f}{\partial p_i} \right).$$

In the early nineteenth century, Poisson noticed that the vanishing of $\{f, g\}$ and $\{f, h\}$ imply that of $\{f, \{g, h\}\}$; almost thirty years later Jacobi discovered the identity $\{f, \{g, h\}\} = \{\{f, g\}, h\} + \{g, \{f, h\}\}$ which "explains" Poisson's theorem. In his study of general composition laws satisfying the Jacobi identity, Lie [29] defined in local coordinate form what is now known as a Poisson structure. On $\mathbf{R}^r$ such a structure is given by functions $w_{ij}(x_1, \cdots, x_r)$ satisfying the identities

$$w_{ij} + w_{ji} = 0,$$

$$\sum_{l=1}^{r} \left(w_{lj}\frac{\partial w_{ik}}{\partial x_l} + w_{li}\frac{\partial w_{kj}}{\partial x_l} + w_{lk}\frac{\partial w_{ji}}{\partial x_l} \right) = 0,$$

Received September 17, 1982. Research supported by the Miller Institute and National Science Foundation Grant MCS 80-23356.