J. DIFFERENTIAL GEOMETRY
18 (1983) 513–521

EMBEDDED HYPERSPHERES WITH PRESCRIBED MEAN CURVATURE

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In [10] Yau raises the nonlinear global problem: is there an embedding $Y: S^n \to \mathbf{R}^{n+1}$ of the n-dimensional sphere into Euclidean $(n+1)$-space, whose mean curvature is a preassigned sufficiently smooth function H defined on $\mathbf{R}^{n+1}$? A theorem of Bakelman and Kantor [4] asserts the existence of such hypersurfaces assuming only natural conditions that H decay faster than the mean curvature of concentric spheres. It is the purpose of this paper to give a new simple geometric treatment of the required a priori estimates and a complete presentation of the existence and uniqueness proof of this result.

A condition that a function H decays in a domain $U \subset \mathbf{R}^{n+1}$-$\{0\}$ from an arbitrary point, say zero, faster than $|X|^{-1}$, where $|X|$ is the Euclidean length of X, is given by

$$(1)\qquad \begin{gathered} 0 < H \in C^1(\bar{U}), \\ \frac{\partial}{\partial \rho} \rho H(\rho X) \leqslant 0, \quad \text{for all } \rho X \in U. \end{gathered}$$

Theorem. (a) *Suppose that the function H satisfies condition* (1) *in the annular region $U = \{X \in \mathbf{R}^{n+1} : r_1 < |X| < r_2\}$ where $0 < r_1 \leqslant 1 \leqslant r_2$ and that*

$$(2)\qquad \begin{aligned} H(x) &> |X|^{-1} \quad \text{for } |X| = r_1, \\ H(X) &< |X|^{-1} \quad \text{for } |X| = r_2. \end{aligned}$$

Then for some $0 < \alpha < 1$ there exists an embedded hypersphere $Y \in C^{2,\alpha}(S^n)$ with mean curvature $\mathfrak{M}Y = H(Y)$ which is a radial graph over the unit sphere such that $r_1 \leqslant |Y| \leqslant r_2$.

(b) *Let Y be a sphere about zero with $\mathfrak{M}Y = H(Y)$. If there is a second embedded C^2 hypersurface Z about zero that satisfies $\mathfrak{M}Z = H(Z)$, and the function H satisfies condition* (1) *in the domain between Y and Z, then the hypersurfaces are homothetic, i.e.,*

$$Z = (1 + t_0)Y, \quad \text{for some } t_0 > -1,$$

Received August 12, 1982, and, in revised form, December 18, 1982. Research partially supported by NSF Grant MCS80-23356.