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FOLIATIONS AND THE TOPOLOGY OF 3-MANIFOLDS

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1. Introduction

Given a compact connected oriented 3-manifold M with boundary ∂M, when does there exist a codimension-1 transversely oriented foliation $\mathfrak{F}$ which is transverse to ∂M and has no Reeb components? If such an $\mathfrak{F}$ exists, then ∂M necessarily is a (possibly empty) union of tori and M is either $S^2 \times S^1$ (and $\mathfrak{F}$ is the product foliation) or irreducible. The first condition follows by Euler characteristic reasons while the latter follows from the work of Rosenberg [24] extending the work of Reeb [23] and Novikov [21]. Our main result says that such conditions are sufficient when $H_2(M, \partial M) \neq 0$.

If such a foliation $\mathfrak{F}$ exists on M, then it follows from the work of Thurston [32] that any compact leaf L is a Thurston norm minimizing surface for the class $[L] \in H_2(M, \partial M)$. Our main result says that for a 3-manifold M satisfying the above necessary conditions any norm minimizing surface can be realized as a compact leaf of a foliation without Reeb components.

Theorem 5.5. *Let M be a compact connected irreducible oriented 3-manifold whose boundary ∂M is a (possibly empty) union of tori. Let S be any norm minimizing surface representing a nontrivial class $z \in H_2(M, \partial M)$. Then there exists foliations $\mathfrak{F}_0$ and $\mathfrak{F}_1$ of M such that:*

(1) *for $i = 0, 1$, $\mathfrak{F}_i \pitchfork \partial M$ and $\mathfrak{F}_i | \partial M$ has no Reeb components,*

(2) *every leaf of $\mathfrak{F}_0$ and $\mathfrak{F}_1$ nontrivially intersects a closed transverse curve,*

(3) *S is a compact leaf of both $\mathfrak{F}_0$ and $\mathfrak{F}_1$,*

(4) *$\mathfrak{F}_0$ is of finite depth,*

(5) *$\mathfrak{F}_1$ is C^∞ except possibly along toral components of S.*

We now state some corollaries of the theorem.

Corollary 6.2. *Let L be an oriented nonsplit link in S^3. Then S is a surface of minimal genus for L if and only if there exists a C^∞ transversely oriented foliation*

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