THE TWISTED INDEX PROBLEM
FOR MANIFOLDS WITH BOUNDARY

PETER B. GILKEY & LANCE SMITH

SECTION ZERO

0.1 Introduction

The Atiyah-Singer index theorem gives a formula in K-theory for the index of any elliptic operator. The Atiyah-Patodi-Singer twisted index theorem [2] is the suspension of the Atiyah-Singer index theorem and gives a measure of spectral flow and spectral asymmetry using the eta invariant with coefficients in a locally flat bundle. It is possible to recover the Atiyah-Singer theorem from the twisted index theorem using certain product formulas so the results of [2] can be viewed as a generalization of the ordinary index theorem as we shall see in §1.3.

It is well known that certain elliptic complexes (for example the signature complex) do not admit local boundary conditions. However, for those which do, the Atiyah-Bott theorem [1] provides a generalization of the index theorem to manifolds with boundary. In a similar fashion, not every twisted index problem admits local boundary conditions; the operator $*d \pm d*$ is one of those which does not as we shall see later. This paper is an effort to combine both the Atiyah-Bott index theorem and the Atiyah-Patodi-Singer twisted index theorem to derive a formula in terms of characteristic classes for the twisted index on a manifold with boundary given local boundary conditions. We are able to treat completely all the operators, arising naturally in Riemannian geometry, which admit local boundary conditions, but the general case is still incomplete although we have a number of strong results in that direction. This formula would contain both the Atiyah-Patodi-Singer twisted index