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AN APPLICATION OF GAUGE THEORY TO FOUR DIMENSIONAL TOPOLOGY

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I.1 Introduction

This paper contains a detailed account of a result in 4-manifold topology, announced in [7], which is proved by analytical and geometrical methods.

It is a consequence of a theorem of J. H. C. Whitehead [22] that the homotopy type of a closed simply connected 4-manifold is entirely determined by the cup-square

$$Q\colon H^2(X;\mathbf{Z}) \to H^4(X;\mathbf{Z}) \cong \mathbf{Z}.$$

If we fix an orientation this becomes a quadratic form on the free Abelian group $H^2(X;\mathbf{Z})$ with determinant ± 1, realised dually on homology as the "intersection form". Many writers have discussed the problem of finding which forms may arise from 4-manifolds of various kinds [11], [12].

Very recently M. H. Freedman has shown [8] that any form of determinant ± 1 may be realised by a simply connected topological 4-manifold. Moreover he proves that the form, together with one extra piece of data (the Kirby-Siebenmann obstruction in $\mathbf{Z}/2$, always zero for smooth manifolds or for even forms) determines the manifold up to homeomorphism.

It has been known for 30 years that some forms cannot be realised by a *smooth* simply connected 4-manifold. We recall that one may divide the forms on the one hand into the even and odd forms (i.e., whether the form takes only even values) and on the other hand into definite and indefinite forms. Then *Rohlin's theorem* asserts that any even form coming from a smooth simply connected 4-manifold has signature divisible by 16. In particular the even definite form E_8 of rank 8 cannot occur in this way. The theorem which we prove here bears instead on the definite forms which we can take without loss of generality to be positive.

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