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STRUCTURE THEOREMS ON RIEMANNIAN SPACES SATISFYING $R(X, Y) \cdot R = 0$. I. THE LOCAL VERSION

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Introduction

The curvature tensor R of a locally symmetric Riemannian space satisfies $R(X, Y) \cdot R = 0$ for all tangent vectors X and Y, where the linear endomorphism $R(X, Y)$ acts on R as a derivation. This identity holds in a space of recurrent curvature also.

The spaces with $R(X, Y) \cdot R = 0$ have been investigated first by E. Cartan [2] as these spaces can be considered as a direct generalization of the notion of symmetric spaces. Further on remarkable results were obtained by the authors A. Lichnerowicz [13], R. S. Couty [3], [4] and N. S. Sinjukov [19], [20], [21]. In one of his papers K. Nomizu [15] conjectered that an irreducible, complete Riemannian space with dim $\geqslant 3$ and with the above symmetric property of the curvature tensor is always a locally symmetric space. But this conjecture was refuted by H. Takagi [22] who constructed 3-dimensional complete irreducible nonlocally-symmetric hypersurfaces with $R(X, Y) \cdot R = 0$. These two papers were very stimulating for the further investigations. We also have to mention the following authors in this field: S. Tanno [23], [24], [25], K. Sekigawa [16], [17] and P. I. Kovaljev [9], [10], [11].

In the following we call a space satisfying $R(X, Y) \cdot R = 0$ a semi-symmetric space. The main purpose of this paper is to determine all semi-symmetric spaces in a structure theorem.

In §1 we give local decomposition theorems using the infinitesimal holonomy group, and in §2 we give some basic formulas. We would like to make it perfectly clear that the results of these chapters are concerning general Riemannian spaces, and not only semi-symmetric spaces. In §3 we construct several nonsymmetric semi-symmetric spaces and in §4 we show that every semi-symmetric space can be decomposed locally on an everywhere dense open subset into the direct product of locally symmetric spaces and of the spaces constructed in §3.

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