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SPECTRAL INVARIANTS OF CONVEX PLANAR REGIONS

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1. Introduction

The inverse spectral problem for planar regions was clearly formulated by M. Kac [9]. In this paper, a summary of which appears in [10], we discuss an approach to this problem, and some limited results, for strictly convex planar domains. The objective of inverse spectral theory is the extraction from the spectrum, say of the Dirichlet problem:

$$\begin{aligned}\Delta u &= \lambda^2 u \quad \text{in } \Omega \subset \mathbf{R}^2,\\ u|_{\partial\Omega} &= 0,\end{aligned} \tag{1.1}$$

of some geometric information about the domain Ω itself. The technique discussed by Kac relies on the fact that the trace of the associated heat equation

$$\tau(t) = \operatorname{tr}(\exp(-t\Delta_D)), \quad t > 0, \tag{1.2}$$

where Δ_D is an unbounded self-adjoint operator on $L^2(\Omega)$, is determined by the spectrum (always with multiplicity):

$$\tau(t) = \sum \exp(-\lambda^2 t). \tag{1.3}$$

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