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THE TOPOLOGY OF FOUR-DIMENSIONAL MANIFOLDS

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To my teachers and friends

0. Introduction

Manifold topology enjoyed a golden age in the late 1950's and 1960's. Of the mysteries still remaining after that period of great success the most compelling seemed to lie in dimensions three and four. Although experience suggested that manifold theory at these dimensions has a distinct character, the dream remained since my graduate school days[1] that some key principle from the high dimensional theory would extend, at least to dimension four, and bring with it the beautiful adherence of topology to algebra familiar in dimensions greater than or equal to five. There is such a principle. It is a homotopy theoretic criterion for imbedding (relatively) a topological 2-handle in a smooth four-dimensional manifold with boundary. The main impact, as outlined in §1, is to the classification of 1-connected 4-manifolds and topological end recognition. However, certain applications to nonsimply connected problems such as knot concordance are also obtained.

The discovery of this principle was made in three stages. From 1973 to 1975 Andrew Casson developed his theory of "flexible handles"[2]. These are certain pairs having the proper homotopy type of the common place open 2-handle $\dot{H} = (D^2 \times \dot{D}^2, \partial D^2 \times \dot{D}^2)$ but "flexible" in the sense that finding imbeddings is rather easy; in fact imbedding is implied by a homotopy theoretic criterion. It was clear to Casson[3] that: (1) no known invariant—link theoretic

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[1] My graduate work was under the direction of William Browder and, informally, Frank Quinn at Princeton University, 1969–1973.

[2] So named by Casson but generally called "Casson handles." We will adhere to the latter, terminology.

[3] See Notes by Guillou on Casson's lectures [15].