J. DIFFERENTIAL GEOMETRY
17 (1982) 139–170

SELF-DUAL YANG-MILLS CONNECTIONS ON NON-SELF-DUAL 4-MANIFOLDS

CLIFFORD HENRY TAUBES

1. The principal results

The purpose of this article is to prove that self-dual Yang-Mills connections exist on a large class of four-dimensional Riemannian manifolds, specifically manifolds with no two dimensional anti-self-dual cohomology.

The differential geometric context is the following. We take M to be a compact connected oriented Riemannian 4-manifold, G a compact connected semi-simple Lie group, and P over M a principal G-bundle. The Yang-Mills functional is defined on the space of smooth connections $\mathcal{C}(P)$ on P as

$$\mathcal{YM}(A) = \frac{1}{2}\int_M |F_A|^2 = \frac{1}{2}\|F_A\|_{L_2}^2, \tag{1.1}$$

where F_A is the curvature of A, and (1.1) is a norm defined in terms of the Riemannian metric on M and the Cartan metric on the Lie algebra $\mathfrak{g}$ of G. This functional has been the subject of recent investigations by many authors in particular, [1], [11], [23].

The critical points of $\mathcal{YM}(\cdot)$ on $\mathcal{C}(P)$ are called Yang-Mills connections. A critical point $A \in \mathcal{C}(P)$ is distinguished by having harmonic curvature in the sense that

$$D_A^{\natural} F_A = 0 \quad \text{(Yang-Mills equation)}, \tag{1.2a}$$

$$D_A F_A = 0 \quad \text{(Bianchi identities)}, \tag{1.2b}$$

where D_A is the covariant exterior derivative.

Let $\hat{\mathfrak{g}} = P \times_{\mathrm{Ad}_G} \mathfrak{g}$ denote the vector bundle which is associated to P by the adjoint representation. The Hodge duality operator $*$ acts on sections of

Received October 22, 1981. The author was a Junior Fellow, Harvard University Society of Fellows, and supported in part by the National Science Foundation under Grant No. PHY 79-16812. The author warmly acknowledges the suggestions, comments and criticisms of Professors R. Bott, A. Jaffe, T. Parker and K. Uhlenbeck, and gives special thanks to Professor Jaffe for his comments on the first drafts of the manuscript.