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HOLOMORPHIC CURVATURES OF ALMOST KÄHLER MANIFOLDS

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1. Introduction and notation

An almost Kähler manifold is an almost Hermitian manifold (M, g, F) such that:

(1) $$(D_X F')(Y, Z) + (D_Y F')(Z, X) + (D_Z F')(X, Y) = 0,$$

where $F'(X, Y) = g(FX, Y)$, D is the natural metric connection, and X, Y, Z are arbitrary vector fields. In this paper we study holomorphic sectional and bisectional curvatures on an almost Kähler manifold. In particular, we obtain results similar to those given by Bishop, Goldberg and Kobayashi [1], [4] on Kähler manifolds and by Gray [5] on nearly Kähler manifolds.

Throughout the paper, we rely heavily on the following identity [2], [3]:

(2) $$\begin{aligned} &K'(X, Y, \bar{Z}, W) + K'(X, Y, Z, \bar{W}) \\ &\quad = \tfrac{1}{2}F'((D_X F)Y - (D_Y F)X, (D_Y F)X, (D_Z F)W - (D_W F)Z), \end{aligned}$$

from which we obtain

(3) $$\begin{aligned} &K'(X, Y, \bar{Z}, \bar{W}) - K'(X, Y, Z, W) \\ &\quad = -\tfrac{1}{2}g((D_X F)Y - (D_Y F)X, (D_Z F)W - (D_W F)Z), \end{aligned}$$

(4) $$K'(\bar{X}, \bar{Y}, \bar{Z}, \bar{W}) = K'(X, Y, Z, W),$$

(5) $$\overline{K(X, Y, \bar{Z})} + K(X, Y, Z) = -\tfrac{1}{2}\left(D_{(D_X F)Y} - F_{(D_Y F)X}\right)Z,$$

where $K(X, Y, Z)$ is the Riemannian curvature tensor, $\bar{X} = FX$ and $K'(X, Y, Z, W) = g(K(X, Y, Z), W)$.The author would like to thank Professor Dr. R. S. Mishra for suggesting the problem and the useful discussions.

2. Holomorphic curvatures

The holomorphic bisectional curvature $h(X, Y)$ is given by

$$h(X, Y) = \frac{-K'(X, \bar{X}, Y, \bar{Y})}{g(X, X)g(Y, Y)}.$$

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