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CR-SUBMANIFOLDS OF A KAEHLER MANIFOLD. II

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1. Introduction

A submanifold N in a Kaehler manifold $\tilde{M}$ is called a CR-submanifold if (1) the maximal complex subspace $\mathfrak{D}_x$ of the tangent space $T_x\tilde{M}$ containing in T_xN, $x \in N$, defines a differentiable distribution on N, and (2) the orthogonal complementary distributiion $\mathfrak{D}^\perp$ of $\mathfrak{D}$ is a totally real distribution, i.e., $J\mathfrak{D}_x^\perp \subseteq T_x^\perp N$, $x \in N$, where J denotes the almost complex structure of $\tilde{M}$, and $T_x^\perp N$ the normal space of N in $\tilde{M}$ at x.

In the first part of this series, we have obtained several fundamental results for CR-submanifolds. In the present part, we shall continue our study on such submanifolds. In particular, we prove that (a) the holomorphic distribution $\mathfrak{D}$ of any CR-submanifold in a Kaehler manifold is minimal (Proposition 3.9); (b) every leaf of the holomorphic distribution of a mixed foliate proper CR-submanifold in a complex hyperbolic space H^m is Einstein-Kaehlerian (Proposition 4.4); and (c) every CR-submanifold with semi-flat normal connection in $\mathbf{C}P^m$ is either an anti-holomorphic submanifold in some totally geodesic $\mathbf{C}P^{h+p}$ of $\mathbf{C}P^m$ or a totally real submanifold (Theorem 5.11).

2. Preliminaries

Let $\tilde{M}^m$ be a complex m-dimensional Kaehler manifold with complex structure J, and N be a real n-dimensional ($n \geqslant 2$) Riemannian manifold isometrically immersed in $\tilde{M}^m$. We denote by $\langle\, ,\rangle$ the metric tensor of $\tilde{M}^m$ as well as that induced on N. Let ∇ and $\tilde{\nabla}$ be the covariant differentiations on N and $\tilde{M}$ respectively. Then the Gauss and Weingartan formulas for N are given respectively by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{2.1}$$

$$\tilde{\nabla}_X \xi = -A_\xi X + D_X \xi, \tag{2.2}$$

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