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THE UNIFORMIZATION OF COMPACT KÄHLER SURFACES OF NEGATIVE CURVATURE

B. WONG

1. Introduction

One of the major accomplishments in the theory of Riemann surfaces is the uniformization theorem which roughly says that the universal covering of a compact Riemann surfaces of genus greater than one is analytically equivalent to the unit disc $\{z \mid |z| < 1\}$. The higher dimensional analog is one of the central problems in hyperbolic complex analysis. In this section we summarize one direction of this research recently moved forward by differential geometers. The starting point for us is a theorem of H. Wu [30] given below.

Theorem 1.1. *Let M be a compact complex Kähler manifold of nonpositive sectional curvature. Then its universal covering is a Stein manifold.*

For a long time, examples of compact complex Kähler manifolds of negative sectional curvature known to us were only compact quotients of the unit ball in C^n, until recently Mostow and Siu discovered a compact Kähler surface of negative sectional curvature which is not uniformized by the ball [20]. A perhaps more natural and nontrivial generalization of hyperbolic Riemann surfaces for algebraic geometers and complex analysis is the notion of negative tangent bundle in the sense of H. Grauert.

Definition. Let M be a compact complex manifold. The tangent bundle $T(M)$ of M is said to be negative if it is a strongly pseudo-convex manifold whose only exceptional variety is the zero section.

The concept of negative tangent bundle is intimately related to that of bisectional curvature described below (see [8], [6]). Let M be a Kähler manifold, and R its Riemannian curvature tensor. Given two complex planes σ and σ' in $T_p(M)$, $p \in M$, we define the bisectional curvature $H(\sigma, \sigma')$ by $H(\sigma, \sigma') = R(X, JX, Y, JY)$, where J is the complex structure tensor of M, $X \in \sigma$, $Y \in \sigma'$. Furthermore, by Bianchi identity we have the following relation

$$R(X, JX, Y, JY) = R(X, Y, X, Y) + R(X, JY, X, JY).$$

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