J. DIFFERENTIAL GEOMETRY
16 (1981) 347–349

FOLIATIONS ON A SURFACE OF CONSTANT CURVATURE AND THE MODIFIED KORTEWEG–DE VRIES EQUATIONS

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Dedicated to Professor Buchin Su on his 80th birthday

ABSTRACT. The modified *KdV* equations are characterized as relations between local invariants of certain foliations on a surface of constant Gaussian curvature.

Consider a surface M, endowed with a C^∞-Riemannian metric of constant Gaussian curvature K. Locally let e_1, e_2 be an orthonormal frame field and ω_1, ω_2 be its dual coframe field. Then the latter satisfy the structure equations

$$(1) \qquad d\omega_1 = \omega_{12} \wedge \omega_2, \quad d\omega_2 = \omega_1 \wedge \omega_{12}, \quad d\omega_{12} = -K\omega_1 \wedge \omega_2,$$

where ω_{12} is the connection form (relative to the frame field). We write

$$(2) \qquad \omega_{12} = p\omega_1 + q\omega_2,$$

p, q being functions on M.

Given on M a foliation by curves. Suppose that both M and the foliation are oriented. At a point $x \in M$ we take e_1 to be tangent to the curve (or leaf) of the foliation through x. Since M is oriented, this determines e_2. The local invariants of the foliation are functions of p, q and their successive covariant derivatives. If the foliation is unoriented, then the local invariants are those which remain invariant under the change $e_1 \to -e_1$.

Under this choice of the frame field the foliation is defined by

$$(3) \qquad \omega_2 = 0,$$

and ω_1 is the element of arc on the leaves. It follows that p is the geodesic curvature of the leaves.

We coordinatize M by the coordinates x, t, such that

$$(4) \qquad \omega_2 = Bdt, \quad \omega_1 = \eta dx + Adt, \quad \omega_{12} = udx + Cdt,$$

Received September 22, 1981. The first author is supported partially by NSF Grant MCS-8023356.