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SUBMERSIONS FROM ANTI-DE SITTER SPACE WITH TOTALLY GEODESIC FIBERS

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Introduction

In [5] O'Neill introduced the notion of a Riemannian submersion. Escobales [1], [2] classified Riemannian submersions from a sphere S^n and from a complex projective space $\mathbf{C}P^n$ with totally geodesic fibers.

This paper investigates such submersions for an indefinite space form: anti-de Sitter space. It is shown that there is essentially only one submersion from H_1^{2n+1} onto a Riemannian manifold with totally geodesic fibers, and this is the standard one onto a complex hyperbolic space $\mathbf{C}H^n$.

1. Let M, B be C^∞ indefinite Riemannian manifolds. An indefinite Riemannian submersion $\pi: M \to B$ is an onto, C^∞ mapping such that

(1) π is of maximal rank,

(2) π_* preserves the lengths of horizontal vectors, i.e., vectors orthogonal to the fibers $\pi^{-1}(x)$, $x \in B$,

(3) the restriction of the metric to the vertical vectors is nondegenerate.

Consider the following example, [4, p. 282, Example 10.7] $p: H_1^{2n+1} \to \mathbf{C}H^n$, where H_1^{2n+1} is a $(2n+1)$-dimensional anti-de Sitter space with constant sectional curvature -1 and signature $(1, 2n)$, and $\mathbf{C}H^n$, defined below, is a complex hyperbolic space. On $\mathbf{C}^{n+1}$ let

$$(\vec{z}, \vec{w}) = -z_0\bar{w}_0 + \sum_{k=1}^{n} z_k\bar{w}_k,$$

$$\langle \vec{z}, \vec{w} \rangle = Re(\vec{z}, \vec{w}) = -x_0u_0 - y_0v_0 + \sum_{k=1}^{n} x_ku_k + y_kv_k,$$

where

$$\vec{z} = (z_0, \cdots, z_n) = (x_0 + iy_0, \cdots, x_n + iy_n),$$

$$\vec{w} = (w_0, \cdots, w_n) = (u_0 + iv_0, \cdots, u_n + iv_n),$$

$$H_1^{2n+1} = \{\vec{z} \in \mathbf{C}^{n+1}: (\vec{z}, \vec{z}) = -1 = \langle \vec{z}, \vec{z} \rangle\}$$
$$= \{(x_0, y_0, \cdots, x_n, y_n): -x_0^2 - y_0^2 + x_1^2 + \cdots + x_n^2 + y_n^2 = -1\}.$$

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