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CR-SUBMANIFOLDS OF A KAEHLER MANIFOLD. I

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1. Introduction

Let $\tilde{M}$ be a Kaehler manifold with complex structure J, N a Riemannian manifold isometrically immersed in $\tilde{M}$, and $\mathfrak{D}_x$ the maximal holomorphic subspace of the tangent space T_xN of N. If the dimension of $\mathfrak{D}_x$ is the same for all x in N, $\mathfrak{D}_x$ gives a holomorphic distribution $\mathfrak{D}$ on N.

Recently, A. Bejancu [1] introduced the notion of a CR-submanifold of $\tilde{M}$ as follows. A submanifold N in a Kaehler manifold $\tilde{M}$ is called a CR-submanifold if there exists on N a differentiable holomorphic distribution $\mathfrak{D}$ such that its orthogonal complement $\mathfrak{D}^\perp$ is a totally real distribution, i.e., $J\mathfrak{D}_x^\perp \subseteq T_x^\perp N$.

In this series of papers, we shall obtain some fundamental properties of CR-submanifolds in Kaehler manifolds.

2. Preliminaries

Let $\tilde{M}$ be a complex m-dimensional Kaehler manifold with complex structure J, and N a real n-dimensional Riemannian manifold isometrically immersed in $\tilde{M}$. We denote by $\langle\ ,\ \rangle$ the metric tensor of $\tilde{M}$ as well as that induced on N. Let ∇ and $\tilde{\nabla}$ be the covariant differentiations on N and $\tilde{M}$, respectively. Then the Gauss and Weingarten formulas for N are given respectively by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{2.1}$$

$$\tilde{\nabla}_X \xi = -A_\xi X + D_X \xi \tag{2.2}$$

for any vector fields X, Y tangent to N and any vector field ξ normal to N, where σ denotes the second fundamental form, and D the linear connection, called the normal connection, induced in the normal bundle $T^\perp N$. The second fundamental tensor A_ξ is related to σ by

$$\langle A_\xi X, Y\rangle = \langle \sigma(X, Y), \xi\rangle. \tag{2.3}$$

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