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SYMPLECTIC STRUCTURES ON GRADED MANIFOLDS

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1. Introduction

Mixed Bose-Fermi systems are currently described by using the idea of "superspace". This concept, introduced by Salam and Strathdee [1] in the context of supersymmetry, has undergone several evolutionary phases. One of the fundamental aims to be reached by the different endeavours of generalizing and making rigorous the definition of superspace is the study of its global topological properties, whose relevance in the theory of dynamical systems cannot be overlooked. The difficulties which are encountered in working out such a program are twofold. In the first place it is necessary to give an unambiguous definition of supermanifold or graded manifold. Secondly, the definition must be flexible enough to allow for the presence of an additional structure on the graded manifold, so that a mechanical theory is actually feasible. As far as the first item is concerned, an up-to-date review of the situation together with a proposal for a new and apparently more general definition of supermanifold is given by Rogers [2]. However we find it more convenient to work in the frame of graded manifolds as defined by Kostant [3], since in our opinion their geometrical features are more thoroughly investigated, especially in the light of the requirements made in the second item. Indeed the calculus of exterior forms and cohomology rings as developed in [3] allows to speak of graded symplectic structures on graded manifolds and provides therefore the natural arena for a mechanical theory.

The purpose of the present work is to show that any graded manifold, whose underlying differentiable manifold has a symplectic structure defined by an exact 2-form [4] can itself be endowed with a graded symplectic structure. This result makes therefore rigorous the construction of mechanical models starting from a differentiable manifold (configuration space) and an arbitrary number of odd generators (independent spin variables). More

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