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AUTOMORPHISM GROUPS OF SOME GEOMETRIC STRUCTURES

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1. Introduction

In this paper we shall investigate the gaps of the dimensions of compact classical Lie groups and the gaps of the dimensions of the automorphism groups of some geometric structures.

Let H be a closed subgroup of $O(n)$. In [12], Montgomery and Samelson have shown that dim H cannot fall into the following range if $n \neq 4$:

$$\langle n-1\rangle_{SO} + \langle 1\rangle_{SO} < \dim H < \langle n\rangle_{SO},$$

where $\langle s\rangle_{SO}$ denotes dim $SO(s)$.

We shall generalize this result by proving the following theorems.

Theorem A. *Let $H \subset G$ be a closed subgroup.*

(a) *If $G = O(n)$, then* dim H *cannot fall into any of the following ranges, i.e., there exist gaps*:

$$\langle n-k\rangle_{SO} + \langle k\rangle_{SO} < \dim H < \langle n-k+1\rangle_{SO},$$

where $1 \leqslant k \leqslant D_{SO}(n)$ if $n \geqslant 13$; or $1 \leqslant k \leqslant A_{SO(n)}$ if $n \geqslant 11$.

(b) *If $G = SU(n)$, then there exist gaps*:

$$\langle n-k\rangle_{SU} + \langle k\rangle_{U} < \dim H < \langle n-k+1\rangle_{SU},$$

where $1 \leqslant k \leqslant D_{SU(n)}$ if $n \geqslant 11$; or $1 \leqslant k \leqslant A_{SU(n)}$ if $n \geqslant 9$.

(c) *If $G = U(n)$, then there exist gaps*:

$$\langle n-k\rangle_{U} + \langle k\rangle_{U} < \dim H < \langle n-k+1\rangle_{SU},$$

where $1 \leqslant k \leqslant D_{U(n)}$ if $n \geqslant 11$; or $1 \leqslant k \leqslant A_{U(n)}$ if $n \geqslant 9$.

(d) *If $G = Sp(n)$, then there exist gaps*:

$$(1)\qquad \langle n-k\rangle_{Sp} + \langle k\rangle_{Sp} < \dim H < \langle n-k+1\rangle_{Sp},$$

where $\langle s\rangle_{SU} =$ dim $SU(s)$, *$\langle s\rangle_{U} =$* dim $U(s)$, *and $\langle s\rangle_{Sp} =$* dim $Sp(s)$, *and $D_{SO(n)}$, $D_{SU(n)}$, $D_{U(n)}$ and $D_{SP(n)}$ are the largest values of k for which the above inequalities in* (a), (b), (c) *and* (d) *are meaningful.* (*For notation A_X, $X = SO$, SU, U and Sp see Theorem* C).

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