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TRANSVERSAL HOLOMORPHIC STRUCTURES

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Among the most important global structures which one can introduce into a differentiable manifold are those obtained by requiring that the jacobians of the coordinate transformations belong to a given linear Lie group G. These structures are called (integrable) G-structures. An ordinary differentiable structure is a G-structure where G is the full linear group $GL(n, \mathbf{R})$. Complex analytic manifolds are obtained by $GL(n, \mathbf{C})$-structures. Another structure which has been intensively investigated is that of foliate structures obtained by the subgroups G of the real or complex general linear groups composed of transformations leaving invariant a linear subspace of euclidean space on which the linear group operates (see, for example, Reeb [18] or Kodaira-Spencer [14]).

Geometrically, a (real) foliation is a decomposition of a manifold M into disjoint connected sets $\{L_\alpha\}$ called the leaves of the foliation such that locally they are isomorphic to the family of horizontal lines $\mathbf{R}^n$ in $\mathbf{R}^{n+q}$.

Additional structure may be introduced in the foliation by controlling more carefully the way the leaves are attached. These are denominated transversal structures and were introduced by Haefliger [11].

In the present work we are interested in transversal holomorphic structures, that is, we assume that the leaves are glued together in a complex analytic manner. These are G-structures where $G = H^{n,q}$ is the subgroup of $GL(n + 2q, \mathbf{R})$ consisting of those matrices of the form

$$\begin{pmatrix} A & A' \\ 0 & A'' \end{pmatrix}$$

where $A \in GL(n, \mathbf{R})$, $A' \in M_{2q,n}(\mathbf{R})$, $A'' \in GL(q, \mathbf{C}) \hookrightarrow GL(2q, \mathbf{R})$.

An $H^{n,q}$-structure in a manifold M is then given by a covering of coordinate patches with coordinates $(x^1, \cdots, x^n; z^1, \cdots, z^q) = (x, z)$ such that the changes of coordinates are local diffeomorphisms of $\mathbf{R}^n \times \mathbf{C}^q$ of the form

$$f(x, z) = (f_1(x, z), f_2(z))$$

where f_2 is a holomorphic function of z alone.

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