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PROJECTIVE MAPPINGS AND DISTORTION THEOREMS

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1. Introduction

Distance- and volume-decreasing theorems have been investigated since Ahlfors [1] extended Schwarz's Lemma. In the complex domain, the results were distortion theorems for various holomorphic (see [9]) and even almost-complex mappings [5]. In the real domain, the theorems were obtained for certain classes of harmonic mappings, mainly by Chern [2], Goldberg [2], [6], [7], T. Ishihara [7], Petridis [7] and the present author [6], [8].

Although the notion of a projective change of a linear connection is classical, the notion of a projective mapping has not been investigated until recently. Two different notions were investigated, a weaker one by Yano and S. Ishihara [14] and a stronger by Kobayashi. The former, discussed in §2, requires the preservation of paths, while the latter, discussed in §4, requires, in addition, the preservation of the projective parameters of Whitehead [12].

In a recent paper [10], Kobayashi showed that *projective mappings* of an interval into a *Riemannian manifold whose Ricci curvature is negative and bounded away from zero are distance decreasing* up to a constant. This is generalized in §5 for mappings of a *complete Riemannian manifold whose Ricci curvature is bounded below*. In particular, this is valid for the hyperbolic open ball, which is the n-dimensional analog of Kobayashi's interval.

For projective mappings in the sense of Yano, we prove in §3 a volume-decreasing theorem, in the equidimensional case, under the same curvature requirements as above. We also show that the two notions of a projective mapping agree if the mapping is a diffeomorphism.

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2. Projective mappings and transformations

Let (M, ∇) and (M', ∇') be differential manifolds with symmetric linear connections. A curve $\gamma : I \to M$ with velocity vector $\dot{\gamma}$ is mapped by a

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