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THE FIRST PROPER SPACE OF Δ FOR p-FORMS IN COMPACT RIEMANNIAN MANIFOLDS OF POSITIVE CURVATURE OPERATOR

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Introduction

Let M^n be an n-dimensional Riemannian manifold, and denote the curvature tensor of M^n by $R_{kji}{}^h$. If there exists a positive constant k such that

$$(*) \qquad -R_{kjih}u^{kj}u^{ih} \geqslant 2ku_{ji}u^{ji}$$

holds for any 2-form u on an M^n everywhere, then the M^n is said to be of positive curvature operator. For a compact orientable M^n of positive curvature operator, M. Berger [1] and D. Meyer [9] have proved that its first $n-1$ Betti numbers $b_i(M^n)$, $i = 1, \cdots, n-1$, vanish. It has been also known that such a manifold is of constant curvature if its metric satisfies $\nabla_h R_{kji}{}^h = 0$, [10]. Let Δ denote the Laplacian operator. A nonzero p-form u satisfying $\Delta u = \lambda u$ with a constant λ is called a proper form of Δ corresponding to the proper value λ. S. Gallot and D. Meyer have discussed the proper value in compact M^n of positive curvature operator and obtained its lower bound as follows.

Theorem A, [6]. *In a compact Riemannian manifold M^n of positive curvature operator, the proper value λ of Δ for p-form u ($n \geqslant p \geqslant 1$) satisfies*

$$\lambda \geqslant p(n-p+1)k \qquad \text{if } du = 0,$$
$$\lambda \geqslant (p+1)(n-p)k \quad \text{if } \delta u = 0.$$

Furthermore, Gallot [2], Gallot and Meyer [7] and the present authors [11], [14] discussed the case when λ actually takes the possible minimal values. In particular, the present authors showed that the Killing and the conformal Killing p-forms play essential roles in this field. On the other hand, one of the present authors has obtained

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