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CONVEX MANIFOLDS OF NONNEGATIVE CURVATURE

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During the past decade, exciting breakthroughs have occurred in the study of complete open (i.e., noncompact) manifolds of nonnegative curvature. Cheeger and Gromoll [3] have shown that such a manifold M contains a compact totally geodesic submanifold S, the *soul* of M, whose normal bundle is diffeomorphic to M itself. The soul also has the property of being convex in the sense of definition 1.1. In fact, the existence of large convex sets with boundary is of basic importance in the construction of the soul. It is mainly because of this that we undertake to better understand their structure, though most questions relating to convexity seem to be important also in their own right.

We first study compact convex sets in the abstract, what we call convex manifolds. We show that such a manifold of nonnegative curvature has a complete metric of nonnegative curvature on its interior. This is used to answer the fundamental question: Can a compact convex manifold of nonnegative curvature be isometrically imbedded into a complete open manifold of nonnegative curvature?–a converse to the procedure of Cheeger and Gromoll. For the most part, but not always, we find the answer to be yes, the most general results being obtained in the case of surfaces.

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1. Convex manifolds

Given a Riemannian manifold N (without boundary) and a set $C \subset N$, C is said to be *convex* if, for any point $p \in \bar{C}$, there is a number $e(p)$ with $0 < e(p) < r(p)$ such that $C \cap B_{e(p)}(p)$ has the property that between any

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