

# SYMMETRY GROUPS AND GROUP INVARIANT SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

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### 1. Introduction

The application of the theory of local transformation groups to the study of partial differential equations has its origins in the original investigations of Sophus Lie. He demonstrated that for a given system of partial differential equations the Lie algebra of all vector fields (i.e., infinitesimal generators of local one-parameter groups transforming the independent and dependent variables) leaving the system invariant could be straightforwardly found via the solution of a large number of auxiliary partial differential equations of an elementary type, the so-called “defining equations” of the group. The rapid development of the global, abstract theory of Lie groups in the first half of this century neglected these results on differential equations for two main reasons: first the results were of an essentially local character and secondly, except for the case of ordinary differential equations, the symmetry groups did not aid in the construction of the general solution of the system under

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