J. DIFFERENTIAL GEOMETRY
14 (1979) 7-20

COMPLETENESS OF CURVATURE SURFACES OF AN ISOMETRIC IMMERSION

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Let M be a hypersurface in a euclidean space, let E_p be the null space of the second fundamental tensor of M at $p \in M$, denote by k the minimum value of the dimensions of the vector spaces E_p on M, and let G be the open subset of M on which this minimum occurs. Then it is well known from classical differential geometry that G is generated by k-dimensional totally geodesic submanifolds along which the normal space of M is constant. Moreover, if M is complete, then these generating submanifolds of G are also complete; this fact was proved first by S. S. Chern and R. K. Lashof [1] and later by many other authors.

In 1971 this theorem was generalized by D. Ferus [3] to submanifolds of higher codimension in arbitrary ambient spaces of constant curvature. The present paper is concerned with a further generalization. While in the above case the generating submanifolds of G may be interpreted as curvature surfaces corresponding to the principal curvature 0, now for an arbitrary principal curvature function λ of M the analogous problem will be considered. A first approach to this general situation was made by T. Otsuki [9] and the author [10]. But the proof of the completeness of the generating submanifolds was left until now. For solving this problem we shall modify the ideas of P. Dombrowski [2], who discovered a fundamental relation between Jacobi fields and so-called geodesic forms.

Applying our results to the case, where M is also a space of constant curvature which exceeds that of the ambient space, we can continue B. O'Neill's investigation [7] to obtain a result analogous to the one of B. O'Neill and E. Stiel [8] about spaces of the same constant curvature.

1. Statement of the principal results

Let $f: M \to N$ be an isometric immersion of a Riemannian manifold M into a Riemannian manifold N of constant curvature with $\dim N > \dim M$. Let $\nu(f)$ denote the normal bundle of f, $\nu^*(f)$ its dual, D the canonical covariant

Received December 3, 1976.