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SOME TOPOLOGICAL OBSTRUCTIONS TO BOCHNER-KAEHLER METRICS AND THEIR APPLICATIONS

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1. Introduction

Let M^n be a compact (complex) manifold of complex dimension n. Let L be a line bundle over M^n. Denote by $H^i(M^n, L)$ the i-th cohomology group with coefficients in the sheaf of germs of local holomorphic sections in L, and by K and 1 the canonical line bundle and the trivial line bundle over M^n respectively. The m-genus or the plurigenera of M^n are given by

$$P_m = \dim H^0(M^n, mK) .$$

where $mk = K \otimes \cdots \otimes K$ (m copies). P_1 is also called the geometric genus p_g of M. By the Serre duality theorem:

$$H^i(M^n, L) \cong H^{n-i}(M^n, L^{-1} \otimes K) ,$$

we also have $p_g = \dim H^n(M^n, 1)$. Put

$$g_i = \dim H^i(M^n, 1) .$$

Then g_1 is called the irregularity of M^n, denoted by $\mathfrak{q}$. The arithmetic genus is then given by

$$\mathfrak{a} = 1 - g_1 + g_2 - \cdots + (-1)^n g_n .$$

In particular, if M^n is a surface (we call a compact connected complex surface free from singularities simply a *surface*), $\mathfrak{a} = 1 - \mathfrak{q} + p_g$. It is well-known that $\mathfrak{a}$, $\mathfrak{q}$, P_m are birational invariants.

In the following we denote by τ, χ, b_i and c_i the Hirzebruch signature, the Euler characteristic, the i-th Betti number and the i-th Chern class of M^n respectively. Let $c \in H^{2n}(M^n, Z)$ be a $2n$-th cohomology class of M^n. We shall also regard c as the integer obtained from the cohomology class c by taking its value on the fundamental cyclic of M^{2n}.

Let g be a Kaehler metric on M^n. We denote by $R^i_{jk\bar{l}}$, $R_{i\bar{j}}$ and ρ respectively

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