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THE RIGIDITY OF SUSPENSIONS

ROBERT CONNELLY

1. Introduction

We investigate the continuous rigidity of a suspension of a polygonal curve in three-space. The main result is that all such embedded suspensions are rigid. This is motivated by the old result of Cauchy (in 1813) [4] that all strictly convex polyhedral surfaces are rigid and by the conjecture that all embedded polyhedral surfaces are rigid.

We develop here some techniques which we feel are new to the subject of rigidity and apply them to obtain two results: We suppose that Σ is a suspension which flexes such that the distance between the suspension points changes, but of course all the edges have a constant length.

Theorem 1. *The winding number of the curve (equator) about the line through the suspension points is zero (when defined).*

For any polyhedral closed (orientable) surface in $\boldsymbol{R}^3$ it is possible to define the notion of a generalized volume, which is defined even if the surface is not embedded but only piecewise linearly mapped into $\boldsymbol{R}^3$. It agrees with the ordinary definition of the volume enclosed by the surface, when the surface is embedded.

Theorem 2. *For Σ as above, $V(\Sigma) = 0$, where $V(\Sigma)$ is the generalized volume.*

Theorem 2 implies that all embedded suspensions are rigid.

Recall from Gluck [6] that a polyhedron P regarded as a simplicial map, linear on each simplex, into $\boldsymbol{R}^3$ is *rigid* iff any homotopy P_t fixing the edge lengths (we call this a *flex*) is congruent in $\boldsymbol{R}^3$ to $P_0 = P$.

The proofs of the above involve first defining certain structural equations which describe the affine algebraic "variety" of the space of congruence classes of isometric maps of the polyhedral surface. This variety is described by certain extrinsic (variable) and intrinsic (constant) parameters, and in a sense Theorems 1 and 2 are formal consequences of the conditions of flexibility. The generalized volume and the winding number are particularly easy to analyse in the way we have chosen to set up the structural equations.

The analysis is based on the observation that the variety can be complexified in a natural way, and since we are interested when the polyhedron flexes we

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