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DERIVATIVES OF SECONDARY CHARACTERISTIC CLASSES

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Introduction

Secondary characteristic classes have been studied extensively in recent years, particularly with regard to foliations. One of the most interesting properties of these classes is their ability to vary continuously with a continuous deformation of the foliation. In this paper we construct the derivatives of these secondary classes for a given foliation.

Let F be a foliation of codimension q on a manifold M. Let Φ be the sheaf of germs of vector fields on M which preserve F. Then $H^1(M; \Phi)$ is the space of infinitesimal deformations of F. There are a graded differential complex WO_q and a natural map

$$\alpha_F^*: H^*(WO_q) \to H^*(M; R)$$

depending only on F, which gives characteristic classes for the foliation. We construct a natural map

$$D_F: H^1(M; \Phi) \times H^*(WO_q) \to H^*(M; R)$$

which depends only on F. This map gives the derivatives of the characteristic classes for the foliation in the sense that if $\beta \in H^1(M; \Phi)$ is the infinitesimal deformation associated to an actual deformation F_s, $s \in R$, $F_0 = F$, then for $f \in H^*(WO_q)$

$$D_F(\beta, f) = \frac{\partial}{\partial s} \alpha_{F_s}^*(f)\Big|_{s=0} .$$

In a crude sense, if one views $\alpha^*(f)$ as a map from the space of foliations on M to the cohomology of M, one may think of $D(.,f)$ as the induced map on the tangent space of the space of foliations. The point of this construction is that it allows one to compute derivatives of characteristic classes corresponding to deformations of a fixed foliation F using only information provided by the foliation, i.e. one does *not* need to know what the deformation is in order to

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