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HOMOTOPICAL EFFECTS OF DILATATION

MIKHAEL GROMOV

1. Statement of results

1.1. Geometrical and topological complexity. Let V and W be Riemannian manifolds, and X a space of mappings $V \to W$. For instance, X may consist of all smooth maps, or may be the space of imbeddings or immersions. We ask how to estimate a measure of the "topological complexity" of an $x \in X$ by geometry of x. We measure geometrical complexity of x by a positive functional $F: X \to \boldsymbol{R}_+$, say, by the dilatation of x or by an integral characteristic like the Dirichlet functional. The topological complexity of x may be measured by its degree (when the degree makes sense) or another numerical invariant.

The Morse theory suggests a different point of view. We take the levels $X_\lambda \subset X$, $X_\lambda = F^{-1}([0, \lambda])$, $\lambda \in \boldsymbol{R}_+$ and compare the numerical invariants of X_λ (say the number of components or the sum of all Betti numbers) with λ.

When $\lambda \to \infty$, the first asymptotic term of the topological complexity of X_λ is often independent of the particular choice of metrics in V and W (but depends, of course, on the particular type of F), and we come to a pure topological problem: how to express this asymptotic topology of X_λ in terms of usual invariants? When we study the asymptotic distribution of the critical values of F, what we need first is the asymptotic behavior of the Betti numbers $b_i(X_\lambda)$, $i, \lambda \to \infty$.

When we seek finer geometro-topological relations in X_λ depending on individual features of V and W, we enter a completely different field resembling geometry of numbers (such as minima of quadratic forms, packing $\boldsymbol{R}^n$ by balls, etc.).

This paper has a definite topological bias.

1.2. The number N of the homotopy classes and the homological dimension ***dm*****.** We denote by $N(\lambda)$ the number of connected components of X intersecting X_λ, where $X_\lambda = F^{-1}([0, \lambda]) \subset X$.

We denote by dm (λ) the maximal integer d such that every map of an arbitrary d-dimensional polyhedron into X is homotopic to a map into X_λ.

1.3. Spectrum of the Laplacian. Consider, for example, the case when W is the real line and X is the projective space associated to the linear space of

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