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REMARKS ON UNIFORMLY EXPANDING HOROCYCLE PARAMETERIZATIONS

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Dedicated to the memory of Rufus Bowen

1. The analysis of Anosov flows on manifolds depends on the analysis of the stable and unstable foliations associated with them, and these foliations have interesting dynamical properties in their own right. In case these foliations are one-dimensional, they give rise to flows, generalizing the horocycle flows on surfaces of negative curvature. B. Marcus[1] has shown that these horocycle flows admit reparameterizations with especially nice properties, which he exploited in proving unique ergodicity. However, he proved that the resulting systems were smooth only rarely, forming closed, nowhere dense sets in appropriate classes of flows. In this note we show that these sets consist essentially of single points. Namely, we prove

Theorem A. *Let $\{f_t\}$ be a C^2 Anosov flow on a compact connected three-dimensional manifold M with stable and unstable orientable foliations W^s and W^u respectively. If W^s and W^u admit C^2 uniformly expanding and contracting parameterizations, then M supports the structure of a homogeneous space of a Lie group with the flow and foliations induced by one-parameter subgroups.*

The three-dimensional G-induced flows which are Anosov are known: they occur in the classification in [1, Chapter III], and are either constant time suspensions of hyperbolic toral automorphisms or are generalized geodesic flows (i.e., flows finitely covered by geodesic flows on tangent bundles of surfaces of constant negative curvature.)

If the flow on M was actually the geodesic flow in the unit tangent bundle of a surface S of negative curvature, Theorem A fails to say much directly about S itself, even though M turns out to be very special. However, it develops that in this case we need only assume one of the horocycle flows to be uniformly reparameterizable:

Theorem B. *Let S be a compact C^∞ surface with negative Gaussian curvature K, and H be the vector field (in the unit tangent bundle) of its unit speed expanding horocycle flow. If H admits a uniformly expanding C^2 reparameterization, then K is constant.*

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[1] [6], [7]; see also Bowen and Marcus [2], and the references cited in these papers to work of Margulis and Lifshitz.