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COMFORMALITY AND ISOMETRY OF RIEMANNIAN MANIFOLDS TO SPHERES. II

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1. Introduction

Let M be an n-dimensional ($n \geq 2$) connected smooth Riemannian manifold with positive definite metric g. If a vector field v on M defines an infinitesimal conformal transformation on (M, g), then v satisfies $\mathscr{L}_v g = 2\rho g$ where $\mathscr{L}_v$ denotes the Lie derivative with respect to v, and ρ is a function on M. v defines an infinitesimal homothetic transformation or infinitesimal isometry according as ρ is constant or zero.

In the last decade or so several authors (for exhaustive lists see [7], [9]) have studied conditions for a Riemannian manifold of dimension $n \geq 2$ with constant scalar curvature k to be either conformal or isometric to a sphere. Recently Ackerman and Hsiung [1], Yano and Hiramatu [7], [8] and Amur and Pujar [2] have studied the conditions without putting restrictions on the scalar curvature k such as $\mathscr{L}_v k = 0$, $\mathscr{L}_{D\rho}\mathscr{L}_v k = 0$ or $[v, D\rho]k = 0$, etc. where $D\rho$ is the vector field on M associated with the differential 1-form $d\rho$.

In this paper we consider a metric semi-symmetric connection $\mathring{\nabla}$ on M induced by a smooth function ρ on M, and obtain conditions for M to be conformal or isometric to a sphere. It is shown in § 5 that our results include some results of Yano and Obata [9] and some of Hsiung and Mugridge [3] as special cases.

2. Notation and formulas

Let ∇ denote a Riemannian connection on M. If x^i, $i = 1, 2, \cdots, n$, are local coordinates in a neighborhood of a point x of M, then the Christoffel symbols associated with ∇ are denoted by $\left\{ {i \atop j\ k} \right\}$, and the components of g by g_{ij}. The raising and lowering of the indices are as usual carried out respectively with g^{ij} and g_{ij}. Let ρ be a smooth function of M. Then $\pi = d\rho$ is *a* smooth closed differential 1-form on M. The local components of π will be denoted by ρ_i. A

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