J. DIFFERENTIAL GEOMETRY
13 (1978) 109-138

ON THE PRINCIPLE OF UNIFORMIZATION

R. S. KULKARNI

The uniformization theorem for Riemann surfaces is a milestone in the classical function theory which has led to several developments in different branches of mathematics. In topology one usually associates the concept of a covering space as directly arising from uniformization. While this is undoubtedly true the classical uniformization theory has a different group theoretical aspect which goes substantially beyond the usual covering space theory. What we have in mind is the special role played by the group of Möbius transformations in the classical uniformization. In several respects it comes close to the more elementary idea of "development" used in differential geometry, and in this form it seems worthwhile to generalize it in other situations. In the classical case there are two broad classes of groups appearing in uniformization. The first class is the class of Fuchsian groups which act discontinuously on the unit disc. The second class is that of Kleinian groups which act discontinuously on some region on the 2-sphere. Even in this classical case it is only in the last decade that Kleinian groups have been vigorously studied thanks to Ahlfors, Bers, Maskit, Kra and others. Higher dimensional generalization of Fuchsian groups have been studied in considerable detail as they arise naturally in the arithmetic problems and the modulii problems. On the other hand the Kleinian case has received much less attention. The method of Kleinian groups leads to construction of new manifolds with far more complicated topology than those arising in the Fuchsian case. Relating the properties of these manifolds to those of the corresponding model spaces and groups is a problem of intrinsic interest.

Here is a brief outline of the contents of the paper.

In § 1 we develop the theory of uniformization. It is implicit in certain classical problems. For instance as mentioned above it appears in its most elementary form in the discussion of developable (i.e., curvature zero) surfaces, and more generally in the theory of space forms (i.e., Riemannian manifolds of constant curvature). In a deeper way it appears in the conformal development of Riemann surfaces. In differential geometry it is implicit in the concept of holonomy especially in the case of integrable G-structures. The author's motivation mainly came from Kuiper [12], [13] and Gunning [7]. The problem of uniformization is the same as that of studying special coordinate coverings proposed by

Communicated by D. C. Spencer, March 31, 1976. Partially supported by the NSF grant GP 32843.